Problem 0: Identification
(1P)
Write your name on each sheet of paper you are using.

Problem 1: Graphing a function
(9P)
Consider the function \( f \) given by
\[
    f(x) := \left| x + \frac{1}{x} \right|.
\]

- Determine the domain of \( f \), i.e., find out for which \( x \)-values the function \( f \) is defined.
- Show that the graph of \( f \) is symmetric with respect to the \( y \)-axis.
- Determine all asymptotes of \( f \) (horizontal, vertical, slant).
- Determine all local minima and maxima of \( f \).
- Determine where \( f \) is increasing or decreasing.
- Determine where \( f \) is concave upward or downward.
- Sketch the graph of \( f \).
Problem 2: Limits

Determine if the following limits exist. If not, explain why. If yes, compute the respective value.

\[
\lim_{x \to 1} \frac{x^a - 1}{x^b - 1} \quad (a, b \in \mathbb{R})
\]

\[
\lim_{x \to 1} \frac{3^x - 3}{x - 1}
\]

Problem 3: Integration

\[
\int \frac{2x^2 - 3}{x - 4} \, dx
\]

Evaluate

\[
\int_0^2 \left( \frac{x}{e} \right)^2 \sqrt{\frac{x^3 + 8}{3}} \, dx
\]