Problem 1 (Conservative vector fields) (3P+6P)

Consider the vector field given in $\mathbb{R}^3$ given by

$$\mathbf{F}(x, y) := (yz + e^x \cos(y)) \cdot \mathbf{i} + (xz - e^x \cdot \sin(y)) \cdot \mathbf{j} + xy \cdot \mathbf{k}.$$ 

(a) Show that $\mathbf{F}$ is conservative.

(b) Find a potential function $f$ for $\mathbf{F}$, i.e., a function $f$ satisfying $\mathbf{F} = \nabla f$.

Problem 2 (Green’s Theorem) (6P)

Let $E$ be the ellipse bounded by

$$\partial E := \left\{ (x, y) \in \mathbb{R}^2 : \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \right\}.$$ 

Evaluate the line integral

$$\int_{\partial E} (x^3 + 2y)dx + (4x - 3y^2)dy.$$ 

Problem 3: (Extrema) (5P)

Find the extrema, if any, of the function $F : \mathbb{R}^2 \to \mathbb{R}$ given by

$$F(x, y) := 3x^3 + y^2 + 4y - 9x.$$ 

Good luck, have fun & do not hesitate to ask questions!!!