MAC 2313 Calculus—Analytic Geometry III

Exam #1

Please hand in your solutions by Mar 24, 2008, 9:30 p.m. Solutions that are handed in later will be graded with 0 points.

Include all computations needed to derive your solution.

Problem 1 (Geometry of $\mathbb{R}^3$) (6P)

Consider the lines

$$L_1 : \langle x, y, z\rangle = \langle 1+t, 1+6t, 2t\rangle \quad (t \in \mathbb{R}) \text{ and} \tag{1}$$

$$L_2 : \langle x, y, z\rangle = \langle 1+s, 5+\frac{15s}{2}, -2+3s\rangle \quad (s \in \mathbb{R}). \tag{2}$$

Find the distance between L_1 and L_2.

Problem 2 (Vector functions) (4P)

Consider the curve C given by

$$\mathbf{r}(t) = \langle 3t+3, t^4+4, t^3+3\rangle$$

and the plane

$$E: \; x - \frac{4y}{3} + z - 1 = 0 \quad .$$

At which point on C is the normal plane parallel to E?

Problem 3: (Lagrange multipliers) (6P)

Find the point P on the paraboloid

$$z = \frac{x^2+y^2}{4} - 2$$

that is closest to the point $Q(0,1,0)$.

Problem 4: (Double integrals) (4P)

Let D be the region in the (x, y)-plane bounded by the x-axis, the line $x = 1$ and the line $y = x$. Compute

$$\iint_D 2 \cdot e^{1-x^2} dA \quad .$$

Good luck !!!