Problem 1 Find a solution of the differential equation

\[ y' = \frac{y \cdot \cos(x)}{1 + y^2} \]

satisfying the initial condition \( y(0) = 1 \).

Problem 2 Is there a function \( y \) satisfying \( y(0) = 2 \) and

\[ \frac{y'}{2x} = \sqrt{1 - y^2} \]?

—Justify your answer. What happens if we replace the initial condition with \( y(0) = 0 \)?

Problem 3 Find the general solution of the differential equation

\[ y' - \cos(x) = -y \cdot \cos(x) \]

Problem 4 Solve the differential equation

\[ y' + y = x \cdot y^3 \]

Hint: Try a substitution \( u = y^\alpha \) with a suitable constant \( \alpha \).