Let $E$ be an elliptic curve given by a Weierstrass equation

$$y^2 = x^3 + Ax + b,$$

and let

$$\alpha(x, y) = \left( \frac{p(x)}{q(x)}, y, \frac{s(x)}{t(x)} \right)$$

be an endomorphism of $E$. Here $p, q, s, t \in K[x]$ are polynomials over the field $K$ with $p, q$ having no root in common and $s, t$ having no root in common.

**Problem 1** Show that

$$\frac{(x^3 + Ax + B)s(x)^2}{t(x)^2} = \frac{u(x)}{q(x)^3}$$

for some $u(x) \in K[x]$ such that $q$ and $u$ have no common root.

*Hint:* You may like to use that a common root of $u$ and $q$ is root of $p$.

**Problem 2** Let $(x_0, y_0) \in E(K)$ such that $t(x_0) = 0$. Show that $q(x_0) = 0$.

Conclude that whenever $q(x_0) \neq 0$, then $\alpha(x_0, y_0)$ is defined.

*Hint:* The polynomial $x^3 + Ax + B$ has no multiple roots.