Optimal Rankings and Labelings of Graphs
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Given a graph $G$, a function $f : V(G) \rightarrow \{1, 2, ..., k\}$ is a $k$-ranking of $G$ if $f(u) = f(v)$ implies every $u - v$ path contains a vertex $w$ such that $f(w) > f(u)$. A $k$-ranking is minimal if the reduction of any label greater than 1 violates the described ranking property. We consider two norms for minimal rankings. In the first norm we minimize the largest label of a minimal ranking and in the second norm we minimize the sum over all labels. The max-optimal norm $\|f(G)\|_\infty$ is the smallest $k$ for which $G$ has a minimal $k$-ranking. This value is also referred to as the rank number $\chi_r(G)$ in previous works. The sum-optimal norm $\|f(G)\|_1$ is the minimum sum of all labels over all minimal rankings. We will investigate similarities and differences between the two norms. In particular we show that paths and cycles that are sum-optimal are also max-optimal.

Finally we will present new results and questions involving other vertex labeling problems.

Keywords: $k$-ranking, vertex labeling, vertex coloring.