Florida Atlantic University
Mathematics Competition

2008–2009

Name: ________________________________

Grade: ________

School: ________________________________

Home Address: ________________________________

____

Phone and/or Email: ____________________

I certify that the work presented herewith
is my own without the help of other people.

Signature: ________________________________

Date: ____________________

Return your solutions with this form as a pdf sent to `xzhang@fau.edu`
or by regular mail (postmarked by January 7, 2009) to

Professor Xiaodong Zhang
Department of Mathematics
Florida Atlantic University
777 Glades Road,
Boca Raton, Florida, 33431-0991

Name:________________

Problem 1.
A company plans to purchase new small cars for $12.15 million. Two models are under consideration, the Sipper for $ 27,000 and the Gulper for $18,000. The estimated annual maintenance is $400 for each model, the budget for maintenance is $ 220,000. The Sipper gets 35 miles per gallon (mpg) while the Gulper gets 24 mpg. The company emphasizes gasoline efficiency, so the numbers x of Sippers and y of Gulpers are to be chosen such that the number $35x + 24y$ is at a maximum.

(a) How many Sippers and Gulpers should the company purchase?

(b) Due to government incentives, there is a $ 2,000 rebate for the Sipper, so its purchase prize is reduced to $ 25,000. How does this change the purchase?

Name:________________

Problem 2.

Call a positive integer normal if it can be written as the difference of two distinct integer squares. What is the 2009-th normal number in the natural number sequence $1, 2, 3, \ldots$?

Name:________________

Problem 3.
ABC is a right triangle of area 6. A', B', C' are the reflections of A in BC, B in CA, C in AB respectively. Find the area of triangle $A'B'C'$.

Name:________________

Problem 4.

Does there exist a positive integer which is doubled when its leftmost digit (in decimal notation) is transferred to the right-most position? If so, find all such numbers. If not, prove the nonexistence.

Name:________________

Problem 5.
ABC is a triangle with $AB : AC = 2 : 1$. If AD is the altitude on BC, show that $AD \leq \frac{2}{3}BC$. Find $\cos A$ when equality holds.

Name:________________

Problem 6.
For a real number x, let $[x]$ denote the greatest integer $\leq x$. Show that for every positive integer n, $[(45+\sqrt{2009})^n]$ is always an odd number.

Name:________________

Problem 7.

There are 8 bus stops between the two ends of a bus route. On a quiet Sunday 11 passengers rode on a bus which went from one end to the other. Show that there are four distinct stops, say, A, B, C, D, so that no passenger gets on the bus at A and leaves at B, same for C and D.